Взаимный базис

$$\mathbf{x} = \sum_{i=1}^{n} c_i \mathbf{u}_i \tag{1}$$

$$(\mathbf{x}, \mathbf{u}_j) = \sum_{i=1}^{n} (\mathbf{u}_i, \mathbf{u}_j) c_i \tag{2}$$

$$(\mathbf{u}_i, \mathbf{v}_j) = \delta_{ij}$$

$$(\mathbf{x}, \mathbf{v}_j) = \sum_{i=1}^{n} c_i (\mathbf{u}_i, \mathbf{v}_j) = c_j \tag{3}$$

$$\mathbf{x} = \sum_{i=1}^{n} (\mathbf{x}, \mathbf{v}_i) \mathbf{u}_i = \sum_{i=1}^{n} (\mathbf{x}, \mathbf{u}_i) \mathbf{v}_i \tag{4}$$

Ортонормированный базис

$(\mathbf{u}_i, \mathbf{u}_j) = \delta_{ij}$

$$(\mathbf{x}, \mathbf{u}_j) = \sum_{i=1}^{n} c_i (\mathbf{u}_i, \mathbf{u}_j) = c_j \tag{5}$$

$$\mathbf{x} = \sum_{i=1}^{n} (\mathbf{x}, \mathbf{u}_i)\mathbf{u}_i \tag{6}$$

$$(\mathbf{x}, \mathbf{y}) = \left(\sum_{i=1}^{n} \alpha_i \mathbf{u}_i, \sum_{j=1}^{n} \beta_j \mathbf{u}_j \right) =$$

$$= \sum_{i=2}^{n} \sum_{j=1}^{n} \alpha_i \overset{*}{\beta}_j (\mathbf{u}_i, \mathbf{u}_j) = \sum_{i=1}^{n} \alpha_i \overset{*}{\beta}_j = (\boldsymbol{\alpha}, \boldsymbol{\beta}) \tag{7}$$

Преобразование Фурье

$$u_0(t) = 1/\sqrt{T}$$
$$u_1(t) = \sqrt{2/T}\sin 2\pi t/T$$
$$u_2(t) = \sqrt{2/T}\cos 2\pi t/T$$
$$\dots$$
$$u_{2m-1}(t) = \sqrt{2/T}\sin 2\pi mt/T$$
$$u_{2m}(t) = \sqrt{2/T}\cos 2\pi mt/T$$
$$\dots$$

$$s(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos 2\pi f_k t + \sum_{k=1}^{\infty} b_k \sin 2\pi f_k t \qquad (8)$$

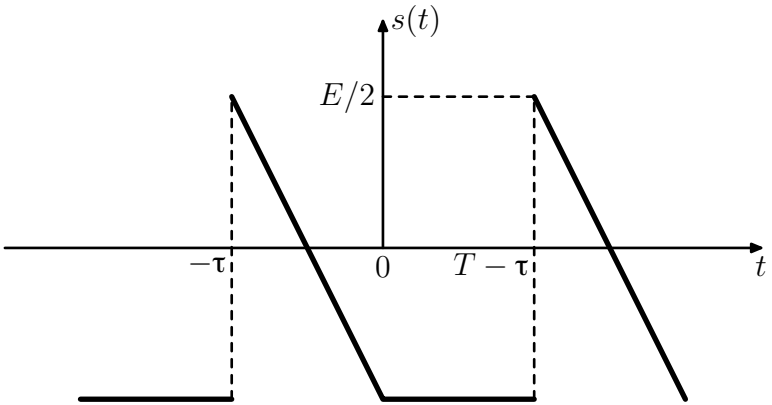
$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos 2\pi f_k t\,dt, \qquad b_k = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\sin 2\pi f_k t\,dt \quad (9)$$

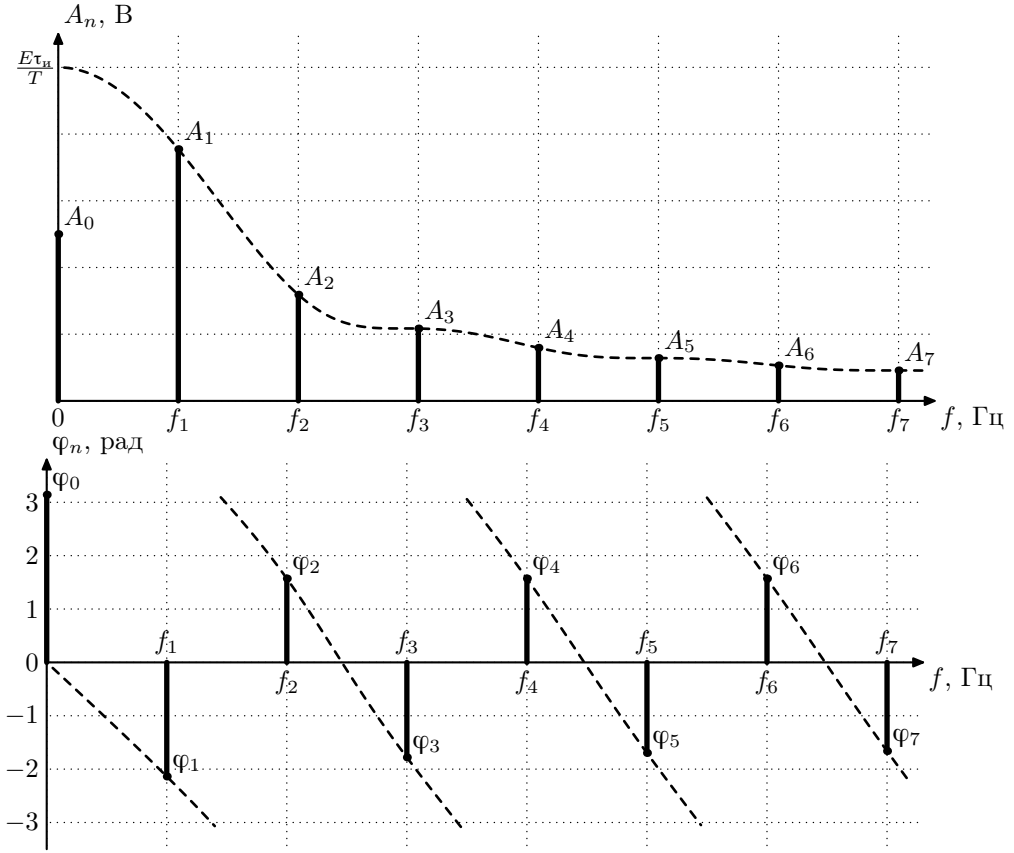
где $f_k = kf_1$ и $f_1 = 1/T$.

$$a_k \cos \omega_k t + b_k \sin \omega_k t = A_k \cos(\omega_k t - \varphi_k) \qquad (10)$$

где $A_k = \sqrt{a_k^2 + b_k^2}, \quad \varphi_k = \operatorname{arctg} \dfrac{b_k}{a_k}.$

$$s(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} A_k \cos(\omega_k t - \varphi_k) \qquad (11)$$




A_n, В
$\frac{E\tau_и}{T}$
A_0
A_1
A_2
A_3
A_4
A_5
A_6
A_7
0
f_1
f_2
f_3
f_4
f_5
f_6
f_7
f, Гц
φ_n, рад
φ_0
3
2
1
0
−1
−2
−3
φ_1
φ_2
φ_3
φ_4
φ_5
φ_6
φ_7
f_1
f_2
f_3
f_4
f_5
f_6
f_7
f, Гц

$$u_k(t) = \frac{1}{\sqrt{T}} \exp\left(j\omega_k t\right) \tag{12}$$

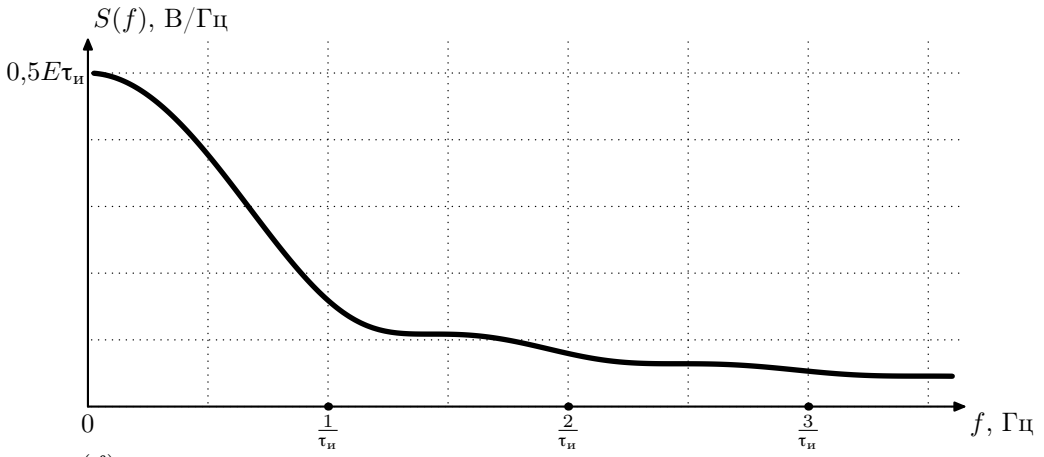
$$s(t) = \sum_{k=-\infty}^{\infty} \dot{C}_k e^{j\omega_k t} \tag{13}$$

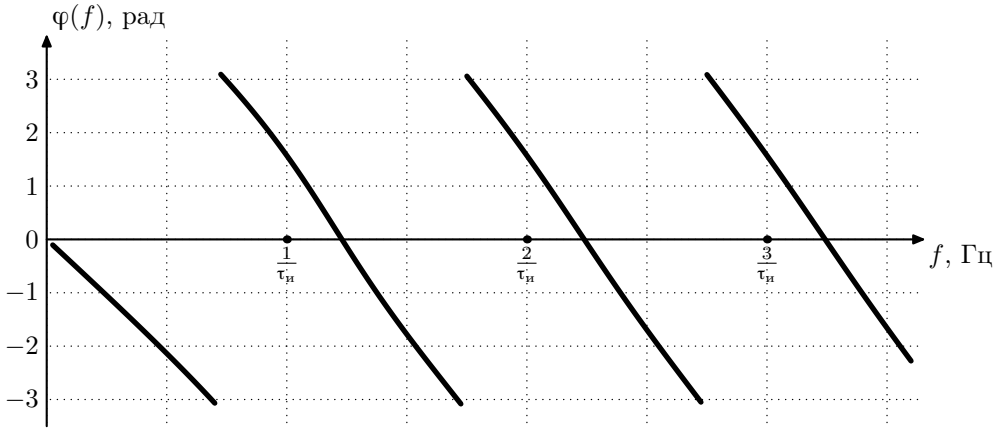
$$\dot{C}_k = \frac{1}{T} \int_{-T/2}^{T/2} s(t) e^{-j\omega_k t}\, dt \tag{14}$$

$$s(t) = \int_{-\infty}^{\infty} \dot{S}(f) e^{j2\pi f t}\, df \tag{15}$$

$$\dot{S}(f) = \int_{-\infty}^{\infty} \dot{s}(t) e^{-j2\pi ft}\, dt \tag{16}$$

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \dot{S}(f) e^{j\omega t}\, d\omega \tag{17}$$


$S(f)$, В/Гц
$0{,}5E\tau_и$
0
$\frac{1}{\tau_и}$
$\frac{2}{\tau_и}$
$\frac{3}{\tau_и}$
f, Гц


$\varphi(f)$, рад
3
2
1
0
−1
−2
−3
$\frac{1}{\tau_и}$
$\frac{2}{\tau_и}$
$\frac{3}{\tau_и}$
f, Гц

Свойства преобразования Фурье

Линейность:

$$\sum_i \alpha_i \dot{s}(t) \leftrightarrow \sum_i \alpha_i \dot{S}(f) \tag{18}$$

Если сигнал s(t) — вещественный, то

$$|\dot{S}(f)| = |\dot{S}(-f)|, \qquad \varphi_s(-f) = -\varphi_s(f) \tag{19}$$

Смещение во времени

$$s(t - t_0) \leftrightarrow \dot{S}(f) e^{-j2\pi f t_0} \tag{20}$$

Изменение масштаба

$$s(kt) \leftrightarrow \frac{1}{k} \dot{S}(f/k) \tag{21}$$

Произведение и свёртка

$$s_1(t)s_2(t) \leftrightarrow \int_{-\infty}^{\infty} \dot{S}_1(\xi)\dot{S}_2(f-\xi)\,d\xi = \dot{S}_1(f) * \dot{S}_2(f) \qquad (22)$$

$$s_1(t) * s_2(t) \leftrightarrow \dot{S}_1(f)\dot{S}_2(f) \qquad (23)$$