Узкополосные сигналы

$$s(t) = A_s(t)\cos\omega_0 t - B_s(t)\sin\omega_0 t \tag{1}$$

$$s(t)\cos\omega_0 t = A_s(t)\cos^2\omega_0 t - B_s(t)\sin\omega_0 t\cos\omega_0 t =$$
$$= 0{,}5A_s(t) + 0{,}5A_s(t)\cos 2\omega_0 t - 0{,}5B_s(t)\sin 2\omega_0 t \tag{2}$$
$$s(t)\sin\omega_0 t = A_s(t)\cos\omega_0 t\sin\omega_0 t - B_s(t)\sin^2\omega_0 t =$$
$$= -0{,}5B_s(t) + 0{,}5A_s(t)\sin 2\omega_0 t + 0{,}5B_s(t)\cos 2\omega_0 t \tag{3}$$

Комплексная огибающая (низкочастотный эквивалент)

$$\dot{s}_{\text{нч}}(t) = A_s(t) + jB_s(t) \tag{4}$$

$$s(t) = \mathrm{Re}(\dot{s}_{\text{нч}}(t)e^{j\omega_0 t}) = A_s(t)\cos\omega_0 t - B_s(t)\sin\omega_0 t \tag{5}$$

$$\dot{s}_{\text{нч}}(t) = U_s(t)e^{j\varphi_s(t)} \tag{6}$$

$$U_s(t) = |\dot{s}_{\text{нч}}(t)| = \sqrt{A_s^2(t) + B_s^2(t)} \tag{7}$$

$$\varphi_s(t) = \mathrm{arctg}\,\frac{B_s(t)}{A_s(t)} \tag{8}$$

$$A_s(t) = U_s(t) \cos \varphi_s(t) \tag{9}$$

$$B_s(t) = U_s(t) \sin \varphi_s(t) \tag{10}$$

$$s(t) = U_s(t) \cos\big(\omega_0 t + \varphi_s(t)\big) = U_s(t) \cos \psi_s(t) \tag{11}$$

$$\psi_s(t) = \omega_0 t + \varphi_s(t) \tag{12}$$

$$\omega_s(t) = \frac{d\psi_s(t)}{dt} = \omega_0 + \frac{d\varphi_s(t)}{dt} \tag{13}$$

$$\dot{S}(\omega) = \int\limits_{-\infty}^{\infty} \operatorname{Re}\left(\dot{s}_{\text{нч}}(t) e^{j\omega_0 t}\right) e^{-j\omega t} dt =$$

$$= {}^1\!/_2 \dot{S}_{\text{нч}}(\omega - \omega_0) + {}^1\!/_2 \overset{*}{S}_{\text{нч}}(-\omega - \omega_0) \qquad \textbf{(14)}$$

Аналитический сигнал

$$s(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\dot{S}(\omega)e^{j\omega t}d\omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{0}\dot{S}(\omega)e^{j\omega t}d\omega + \frac{1}{2\pi}\int_{0}^{\infty}\dot{S}(\omega)e^{j\omega t}d\omega \tag{15}$$

$$\dot{s}_{\text{a}}(t) = \frac{1}{\pi}\int_{0}^{\infty}\dot{S}(\omega)e^{j\omega t}d\omega \tag{16}$$

$$s(t) = {}^1\!/_2\big(\dot{s}_{\text{a}}(t) + \overset{*}{s}_{\text{a}}(t)\big) = \operatorname{Re}\dot{s}_{\text{a}}(t) \tag{17}$$

$$\tilde{s}(t) = \operatorname{Im}\dot{s}_{\text{a}}(t) \tag{18}$$

$$\dot{s}_{\text{a}}(t) = s(t) + j\tilde{s}(t) \tag{19}$$

Преобразование Гильберта

$$\dot{S}_{\text{a}}(\omega) = \begin{cases} 0, & \omega < 0 \\ 2\dot{S}(\omega), & \omega \geqslant 0 \end{cases} \quad (20)$$

$$\dot{S}_{\text{a}}(\omega) = \dot{S}(\omega) + j\tilde{S}(\omega) \quad (21)$$

$$\tilde{S}(\omega) = -j\,\mathrm{sign}(\omega)\dot{S}(\omega) = \begin{cases} j\dot{S}(\omega), & \omega < 0 \\ -j\dot{S}(\omega), & \omega \geqslant 0 \end{cases} \quad (22)$$

$$\tilde{s}(t) = s(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int\limits_{-\infty}^{\infty} \frac{s(\tau)\,d\tau}{t-\tau} \quad (23)$$

$$\dot{S}(\omega) = j\,\mathrm{sign}(\omega)\tilde{S}(\omega) \qquad (24)$$

$$s(t) = -\tilde{s}(t) * \frac{1}{\pi t} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\tilde{s}(\tau)\, d\tau}{\tau - t} \qquad (25)$$

Свойства преобразования Гильберта

$$\mathrm{H}\left[\alpha_1 s_1(t) + \alpha_2 s_2(t)\right] = \alpha_1 \,\mathrm{H}\left[s_1(t)\right] + \alpha_2 \,\mathrm{H}\left[s_2(t)\right]; \qquad (26)$$

$$\mathrm{H}[\alpha] = 0, \qquad \alpha = \mathbf{const}; \qquad (27)$$

$$\mathrm{H}[\cos\omega t] = \mathrm{sign}(\omega)\sin\omega t \qquad (28)$$

$$\mathrm{H}[\sin\omega t] = -\,\mathrm{sign}(\omega)\cos\omega t \qquad (29)$$

Если $s(t) = A_s(t)\cos\omega_0 t - B_s(t)\sin\omega_0 t$, **то**

$$\tilde{s}(t) = H\left[s(t)\right] = B_s(t)\cos\omega_0 t + A_s(t)\sin\omega_0 t \qquad (30)$$

$$U_s(t) = |\dot{s}_{\text{а}}(t)| = \sqrt{s^2(t) + \tilde{s}^2(t)} \tag{31}$$

$$\dot{s}_{\text{нч}}(t) = \dot{s}_{\text{а}}(t)e^{-j\omega_0 t} \tag{32}$$