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Optimum Parametric Synthesis of Broadband Matched Junctions

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Abstract—A method of synthesizing of smooth, broadband matched junctions is proposed that is based on numerical integration of a differential equation for the coefficient of wave reflection from inhomogeneities. Optimum profiles of the wave impedance and frequency dependences of the power reflection coefficient are obtained and analyzed. It is shown that the proposed method allows solution of the synthesis problem to be significantly optimized due to minimization of computation time.

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In radio engineering, it is frequently necessary to solve problems related to the propagation of waves in inhomogeneous media with material parameters dependent on the spatial coordinates [1, 2]. In particular, segments of inhomogeneous transmission lines are widely used in microwave and shortwave devices.

This investigation is devoted to inhomogeneities represented by junctions between two transmission lines with different cross sections. The interest in these junctions is related to the need for connecting two transmission lines with various geometric parameters so as to ensure matching in a maximum possible frequency band [3–6]. As is known, under otherwise equal conditions, the junctions with continuously varying electrical parameters (smooth junctions) ensure a lower level of reflection in a broad frequency band as compared to step junctions [7]. In practice, the junctions with exponential, Chebyshev type, and probabilistic profiles [8] are most frequently employed, the latter representing a limiting case of the step junction with a flat characteristic.

Let us consider a segment of an inhomogeneous microstrip transmission line (Fig. 1). In the region of $0 \le x \le a$, the line is characterized by linear capacitance $C(x)$, inductance $L(x)$, and conductance $G(x)$, the combinations of which determine local values of the wave impedance $Z(x) = \sqrt{L(x)/C(x)}$ and phase velocity $v(x) = 1/\sqrt{L(x)C(x)}$. The corresponding values for homogeneous input and output regions of the line, respectively, will be denoted by Z_0, Z_a and v_0, v_a. Wave propagation in the transmission line segment is described by the telegraph equations [7]:

$$\frac{dU}{dx} = -jkn(x)\rho(x)V,$$
$$\frac{dV}{dx} = -[jkn(x)\rho^{-1}(x) + 2\delta(x)]U, \tag{1}$$

where

$$V(x) = I(x)Z_0, \quad k = \omega/v_0, \quad n(x) = v_0/v(x),$$
$$\rho(x) = Z(x)/Z_a, \quad 2\delta(x) = G(x)Z_a.$$

For an electromagnetic wave with complex amplitude U_0 incident on the inhomogeneous region along the Ox axis, the reflection coefficient is defined as the ratio $R = U_r/U_0$, where U_r is the reflected wave amplitude. It is convenient to introduce normalized variables as $Y_1 = U/U_0$ and $Y_2 = V/U_0$, in which Eqs. (1) can be written in the following form:

$$\frac{dY_1}{dx} = A_{12}(x)Y_2(x), \quad \frac{dY_2}{dx} = A_{21}(x)Y_1(x), \tag{2}$$

where $A_{12} = -jkn(x)\rho(x)$ and $A_{21} = -jkn(x)\rho^{-1}(x) - 2\delta(x)$.

The boundary conditions for Eqs. (2), which correspond to the physical situation under consideration, can be written as follows:

$$Y_1(a) = R + 1, \quad Y_2(a) = R - 1,$$
$$Y_1(0) = T, \quad Y_2(0) = -\frac{T}{\rho_0}, \tag{3}$$

where $T = U_t/U_0$ is the transmission coefficient and $\rho_0 = Z_0/Z_a$ is the wave impedance change. Using boundary conditions (3) for $a = 0$, it is possible to determine the coefficient of reflection from a junction between two lines with wave impedances Z_a and Z_0 as

$$R(0) = \frac{\rho_0 - 1}{\rho_0 + 1}. \tag{4}$$

This relation is used as the initial condition in solving the differential equation for the reflection coefficient [7]:

$$\frac{dR}{d\xi} = \frac{1}{2}[A_{21}(\xi)(1 + R)^2 - A_{12}(\xi)(1 - R)^2], \tag{5}$$

where $\xi = a/x$ is the normalized coordinate.

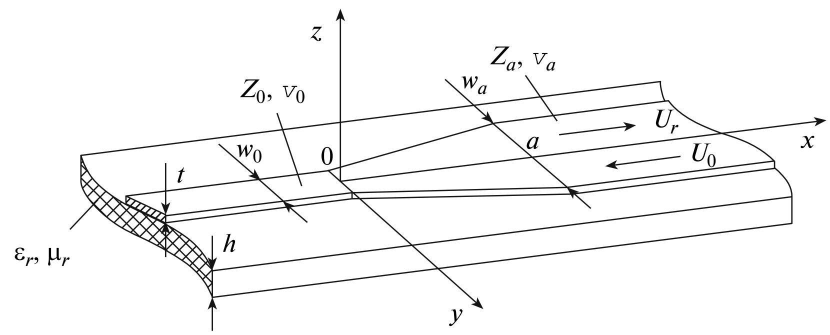

Fig. 1. Segment of an inhomogeneous microstrip transmission line.

If the synthesis does not require determining the wave field distribution in the inhomogeneous region, the reflection coefficient should be calculated by solving the Cauchy problem given by Eqs. (4) and (5). Note that Eq. (5) is a nonlinear equation with variable coefficients; it has to be solved by numerical methods.

Let us introduce a uniform grid with step $h = a/N$ in the region of $0 \le x \le a$, where $N = 1$ is the total number of nodes and the nodes with numbers $n = 0$ and $n = N$ correspond to the left and right boundaries of the inhomogeneous region, respectively. The derivative in Eq. (5) is approximated by the central difference

$$\left.\frac{dR}{d\xi}\right|_n = \frac{R_{n+1} - R_{n-1}}{2h} + O(h^2). \quad (6)$$

In this formula, index n refers to values of the function and its derivative in the nth node (i.e., for $\xi = nh$). Using this approximation, Eq. (5) in the nth node takes the following form:

$$\begin{aligned} R_n = R_{n-2} - hA_{12}^{n-1}(k)(1 - R_{n-1})^2 \\ + hA_{21}^{n-1}(k)(1 + R_{n-1})^2. \end{aligned} \quad (7)$$

An expression for the node with $n = 1$ is written using the values of functions at $\xi_{1/2} = h/2$:

$$\begin{aligned} R_1 &= R_0 \\ &= \frac{h}{2}[A_{12}^{1/2}(k)(1 - R_{1/2})^2 + A_{21}^{1/2}(k)(1 + R_{1/2})^2]. \end{aligned} \quad (8)$$

Here, $R_{1/2}$ is calculated using an approximated derivative according to Eq. (6) with the right-hand-side difference:

$$R_{1/2} = R_0 - \frac{h}{4}[A_{12}^0(k)(1 - R_0)^2 - A_{21}^0(k)(1 + R_0)^2]. \quad (9)$$

Equations (7)–(9) represent a difference scheme of second-order accuracy with respect to the discretization step, which only requires a single calculation of the right-hand part of Eq. (5). Note that the two-stage Runge–Kutta method, which has a comparable accuracy, is not as fast, since it requires double calculation of the right-hand part of Eq. (5).

Synthesis of a wave-impedance profile with smooth junction is formulated as a problem of minimization of the target mismatch function [7],

$$\Phi(\vec{\alpha}) = \sum_{m=1}^{M} b_m \left|R(k_m; \vec{\alpha})\right|^2 \quad (10)$$

on a grid of frequencies k_m in the space of parameters $\vec{\alpha}$ of the wave impedance profile $\rho(x; \vec{\alpha})$, where b_m is the weight coefficients. In calculating the $R(k_m; \vec{\alpha})$ values by difference scheme (7)–(9), the search of minimum of the target function (10) can be performed by a direct-optimization method. The given algorithm can be used for both analysis and synthesis of matched junctions.

As an example of using the proposed method, let us synthesize a piecewise-linear junction comprising a sequence of two linear parts. A spatial profile of the wave impedance dependent on two normalized parameters (ρ_*, a_*) can be described as follows:

$$\rho(x; a_*, \rho_*)$$

$$= \begin{cases} \rho_0 + (\rho_* - \rho_0)\dfrac{x}{a_*} & \text{for } 0 \le x/a \le a_*, \\ \rho_0 + \dfrac{1 - \rho_*}{1 - a_*}\left(\dfrac{x}{a} - a_*\right) & \text{for } a_* \le x/a \le 1. \end{cases} \quad (11)$$

Initial data for the synthesis of a junction between lines can be represented by the frequency dependence of the coefficient of reflection from a parabolic junction:

$$\rho(x) = \rho_0 - 2(\rho_0 - 1)(x/a) + (\rho_0 - 1)(x/a)^2. \quad (12)$$

Figure 2 shows plots of the square moduli of power reflection coefficients versus normalized frequency $K = ka$ for $\rho_0 = 10$ in the case of (*1*) piecewise-linear and (*2*) parabolic wave impedance profiles. As can be seen, the optimized piecewise-linear junction pro-

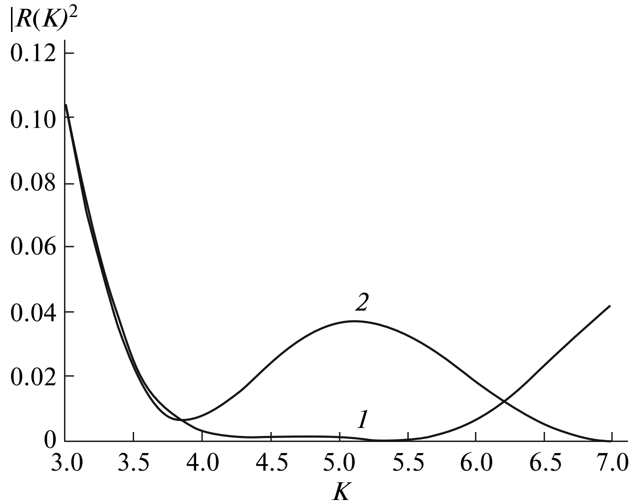

Fig. 2. Frequency dependences of the square moduli of reflection coefficients for junctions with (*1*) piecewise-linear and (*2*) parabolic wave impedance profiles.

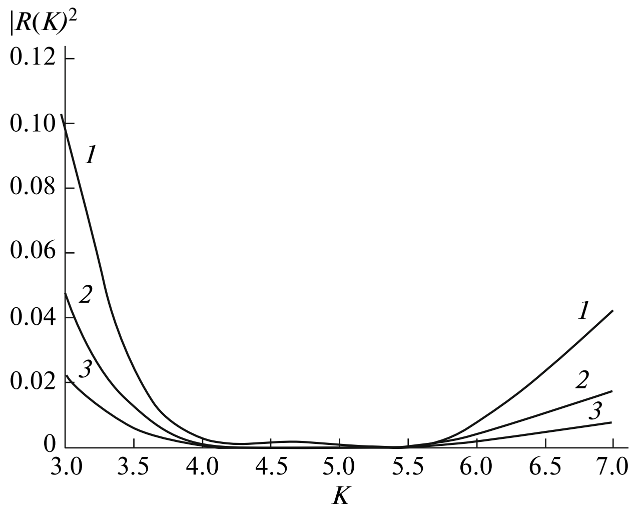

Fig. 3. Frequency dependences of the square moduli of reflection coefficients for synthesized piecewise-linear junctions with various differences of wave impedances ρ = 10 (*1*), 5 (*2*), and 3 (*3*).

vides better matching in a broad frequency range than does the parabolic junction.

Figure 3 presents frequency dependences of the square moduli of reflection coefficients for synthesized piecewise-linear junctions with various differences of wave impedances $\rho = 10$ (curve *1*), 5 (curve *2*), and 3 (curve *3*). As can be seen, the synthesized piecewise-linear junctions ensure good matching in a frequency band corresponding to $4 \leq K \leq 6$.

Thus, we have proposed a fast algorithm for numerical analysis of the frequency characteristics of junctions between transmission lines with different wave impedances. Using a previously obtained [7] equation for the coefficient of reflection from an inhomogeneous region of the transmission line, a method has been developed for the optimum parametric synthesis of broadband matched junctions between transmission lines with different wave impedances. Piecewise-linear junctions have been synthesized, and it has been established that the optimum junction of this type ensures better matching in a broad frequency band than does a higher-order parabolic junction. The proposed method of synthesis of smooth, broadband junctions can be used for designing devices such as matching transformers, filters, coupling elements of transmission lines, etc.

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