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PHYSICS

Kravchenko–Wigner Transformation in Nonlinear Digital Signal Processing

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INTRODUCTION

In the last few years, significant attention has been directed in different fields of science and technology to the application of signals of novel forms. This interest stimulated the development of new mathematical methods for signal processing. In particular, the wider application of ultra-wideband (UWB) signals and their increasing variety (ultrashort [1], directly-chaotic [2], nonlinear [3], fractal [4], and UWB signals) has shown the insufficiency of solely traditional methods of analysis. We imply the methods of the dynamic Fourier transformation and of the Fourier spectrogram. Nowadays, various modern methods of analysis begin to be used successfully. Among them, there are various forms of wavelet transformation (WT) (continuous WT, analytical WT, discrete, stationary WT, wavelet series, wavelet packets) [5] and nonlinear transformations of the Cohen class (Wigner transformation and Choi–William transformation) [6], as well as the atomic-function (AF) theory [7–9].

Each of these transformations along with their doubtless advantages also has certain disadvantages.

In the present study, we propose for the first time a new transformation that combines the advantages of the Wigner transformation and of atomic functions with the decreasing role of their disadvantages.

FORMS OF THE WIGNER TRANSFORMATION

The Wigner transformation of the signal $f(t)$ is given by the relationship [10, 11]

$$P_V[f(t)] \equiv P_V f(\tau, \omega) = \int_{-\infty}^{\infty} f\left(\tau + \frac{t}{2}\right) f^*\left(\tau - \frac{t}{2}\right) \exp(-i\omega t) dt. \quad (1)$$

Here, the asterisk denotes the operation of complex conjugation. The Wigner transformation is quadratic with respect to the process under study (i.e., to the signal $f(t)$). That is why the application of this operation to the sum of two signals is associated with the appearance of an interference term of the form

$$I[f_1, f_2] = P_V[f_1, f_2] + P_V[f_2, f_1].$$

This term is the real-valued function of the variables τ and ω and can be nonzero in the regions where $P_1 f_1$ and $P_1 f_2$ are zero. The appearance of the interference terms results in the detection of false signals.

There exist several different methods that make it possible to reduce the level of the interference terms as well as methods based on averaging the integrand in relationship (1). One of these methods is known [10, 11] as the Choi–William transformation. The other one is the use of the frequency-smoothing window $h(t)$ in the time region. As a result, we deal with the so-called Wigner pseudo-transformation (WPT) [11]

$$P_{PV} f(\omega, \tau; h) = \int_{-\infty}^{\infty} h(t) f\left(\tau + \frac{t}{2}\right) f^*\left(\tau - \frac{t}{2}\right) \exp(-i\omega t) dt. \quad (2)$$

The reduction in the level of the interference terms leads to the deterioration of the frequency resolution. For $h(t) = 1$, the WPT becomes the usual Wigner transformation determined by expression (1). The third method develops the concept of the second one: the time smoothing window $g(t)$ is exploited together with

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the window $h(t)$. The corresponding transformation is of the form

$$P_{SPV}f(\omega, \tau; h, g) = \int_{-\infty}^{\infty} h(t) \int_{-\infty}^{\infty} g(s-\tau) f\left(s+\frac{t}{2}\right) f^*\left(s-\frac{t}{2}\right) ds \, \exp(-i\omega t) dt. \quad (3)$$

It is called the Wigner smoothed pseudotransformation (WSPT). For $g(t) = \delta(t)$, the WSPT becomes the WPT. With improving the interference suppression, the frequency–time resolution deteriorates. We should note that, in the case when both windows $h(t)$ and $g(t)$ are Gaussian ones, the WSPT [11] transforms to the Fourier spectrogram. The advantage of the Wigner transformation compared to the linear-processing methods is its nonlinearity. This allows us to analyze efficiently signals obeying a non-Gaussian distribution law against the background.

WEIGHT FUNCTIONS (WINDOWS) OF KRAVCHENKO AND OF KRAVCHENKO–RVACHEV

It is well known that atomic functions are widely used in various physical applications [7–9]. In this study, among the large variety of existing spectral windows based on atomic functions, we choose the following Kravchenko–Rvachev windows:

$$w_1(t) = \mathrm{up}(t), \quad (4)$$

$$w_2(t) = \frac{\mathrm{fup}_1(3t/2)}{\mathrm{fup}_1(0)}, \quad (5)$$

$$w_3(t) = h_{3/2}(t), \quad (6)$$

$$w_4(t) = \frac{\Xi_2(t)}{\Xi_2(0)}. \quad (7)$$

These windows are constructed on the basis of atomic functions up(t), $\mathrm{fup}_N(t)$, $h_a(t)$, and $\Xi_n(t)$. The advantages of the application of the Kravchenko–Rvachev weight functions (windows) for performing digital signal analysis compared to other windows were shown in [7].

NEW CLASS OF KRAVCHENKO–WIGNER TRANSFORMATIONS

It seems rather reasonable and promising to employ the Kravchenko–Rvachev spectral windows in nonlinear digital signal processing. This approach allows us to combine unique properties of both atomic functions and nonlinear transformations. One of the possible variants for the realization of this approach can be the involvement of the Kravchenko–Rvachev weight functions in the WPT. Substituting expressions (4)–(7) into (2), we arrive at a new type of transformations

$$P_{PKV}f(\omega, \tau; \mathrm{up}) = \int_{-\infty}^{\infty} \mathrm{up}(t) f\left(\tau+\frac{t}{2}\right) f^*\left(\tau-\frac{t}{2}\right) \exp(-i\omega t) dt,$$

$$P_{PKV}f(\omega, \tau; \mathrm{fup}_1) = \frac{1}{\mathrm{fup}_1(0)} \int_{-\infty}^{\infty} \mathrm{fup}_1\left(\frac{3t}{2}\right) f\left(\tau+\frac{t}{2}\right) f^*\left(\tau-\frac{t}{2}\right) \exp(-i\omega t) dt,$$

$$P_{PKV}f(\omega, \tau; h_{3/2}) = \int_{-\infty}^{\infty} h_{3/2}(t) f\left(\tau+\frac{t}{2}\right) f^*\left(\tau-\frac{t}{2}\right) \exp(-i\omega t) dt,$$

$$P_{PKV}f(\omega, \tau; \Xi_2) = \frac{1}{\Xi_2(0)} \int_{-\infty}^{\infty} \Xi_2(t) f\left(\tau+\frac{t}{2}\right) f^*\left(\tau-\frac{t}{2}\right) \exp(-i\omega t) dt.$$

We call the transformations of this type the Kravchenko–Wigner pseudo-transformations (KWPTs). The other realization of the concept given above is the involvement of the Kravchenko–Rvachev windows in the WSPT. Upon substituting expressions (4)–(7) into (3), we arrive at the following transformations:

$$P_{SPKV}f(\omega, \tau; \mathrm{up}, \mathrm{up}) = \int_{-\infty}^{\infty} \mathrm{up}(t) \int_{-\infty}^{\infty} \mathrm{up}(s-\tau) \times f\left(s+\frac{t}{2}\right) f^*\left(s-\frac{t}{2}\right) ds \exp(-i\omega t) dt,$$

$$P_{SPKV}f(\omega, \tau; \mathrm{fup}_{3/2}, \mathrm{fup}_{3/2}) = \frac{1}{\mathrm{fup}_{3/2}^2(0)} \int_{-\infty}^{\infty} \mathrm{fup}_{3/2}(t) \times \int_{-\infty}^{\infty} \mathrm{fup}_{3/2}(s-\tau) f\left(s+\frac{t}{2}\right) f^*\left(s-\frac{t}{2}\right) (ds \exp(-i\omega t)) dt,$$

$$P_{SPKV}f(\omega, \tau; h_{3/2}, h_{3/2}) = \int_{-\infty}^{\infty} h_{3/2}(t) \int_{-\infty}^{\infty} h_{3/2}(s-\tau) \times f\left(s+\frac{t}{2}\right) f^*\left(s-\frac{t}{2}\right) ds \exp(-i\omega t) dt,$$

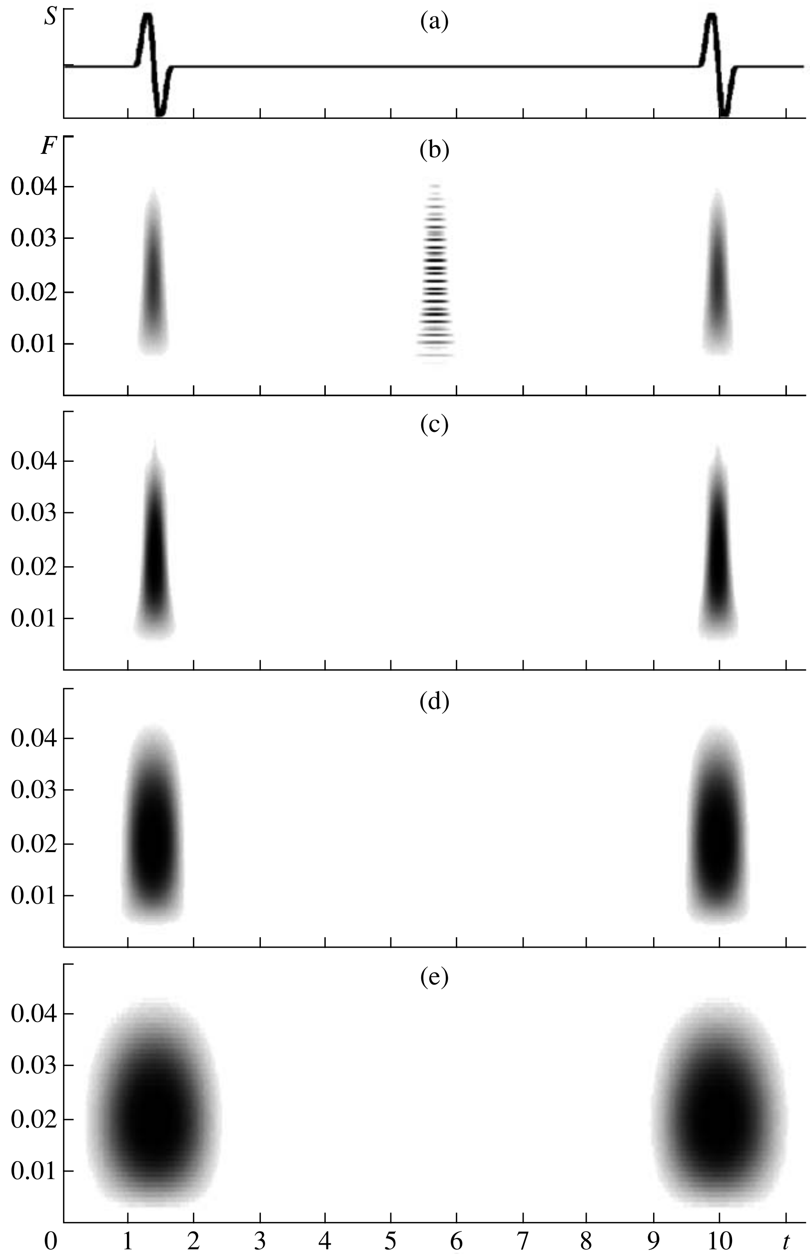

Fig. 1. Two ultra-wideband signals: (a) in the time region; (b) spectral-density function (SDF) for the Wigner transformation; (c) SDF for the Kravchenko–Wigner pseudo-transformation; (d) SDF for the Kravchenko–Wigner smoothed transformation; and (e) the Fourier spectrogram.

$$P_{SPKV}f(\omega, \tau; \Xi_2, \Xi_2) = \frac{1}{\Xi_2^2(0)}\int_{-\infty}^{\infty}\Xi_2(t)\int_{-\infty}^{\infty}\Xi_2(s-\tau) \times f\left(s+\frac{t}{2}\right)f^*\left(s-\frac{t}{2}\right)ds\exp(-i\omega t)dt.$$

We call them the Kravchenko–Wigner smoothed pseudo-transformations (KWSPT). It is self-evident that we can also use in this case combinations of different Kravchenko–Rvachev windows.

DISCUSSION OF THE RESULTS

The results of the application of the new class of Kravchenko–Wigner transformations is demonstrated in Figs. 1 and 2, where the KWPTs and KWSPTs based on the atomic function up(t) are used for analysis of model ultra-wideband signals. In this case, we also observe the suppression of interference terms, which, at the same time, is associated with the reasonably good frequency–time resolution. The KWPTs and KWSPTs, being quadratic with respect to the signal under study,

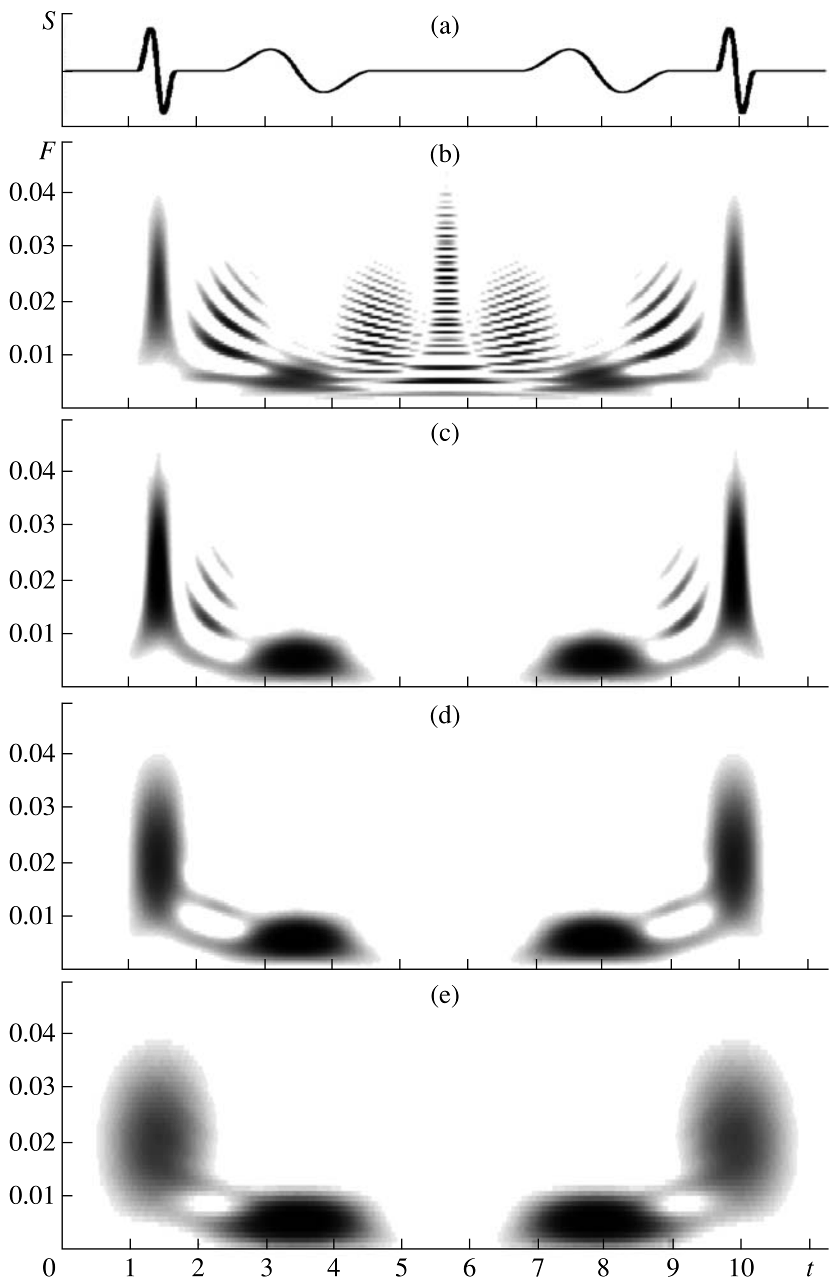

Fig. 2. Four ultra-wideband signals: (a) in the time region; (b) spectral-density function (SDF) for the Wigner transformation; (c) SDF for the Kravchenko–Wigner pseudo-transformation; (d) SDF for the Kravchenko–Wigner smoothed transformation; and (e) the Fourier spectrogram.

can occupy a highly important place among other existing methods of the digital signal processing that have become more and more propagated in recent years. The efficiency of the new class of Kravchenko–Wigner transformations can be manifested, e.g., in the analysis of ultra-wideband signals in the presence of a background. The statistics of background signals noticeably differs from that of normal ones. The linear processing of these events turns out to be inefficient due to the correlation between expansion components. The cost paid for the advantages obtained is a certain deterioration of the frequency–time resolution compared to the usual Wigner transformation (1). However, the frequency–time resolution of the KWPT and KWSPT is better than that provided by Fourier spectrograms.

Thus, the KWPTs and KWSPTs belong to the new class of nonlinear Kravchenko–Wigner transformations. They combine the advantages of both Wigner nonlinear transformations and atomic functions. It is reasonable and promising to apply the KWPTs and

KWSPTs for the analysis of new classes of signals based on atomic functions, *R*-functions, and wavelets. In contrast to classical narrow-band and broadband signals, the signals we deal with are localized in the time region and require the corresponding frequency–time resolution.

REFERENCES

1. L. Yu. Astanin and A. A. Kostylev, *Fundamentals of Super-Broadband Radar Measurements* (Radio i Svyaz', Moscow, 1989) [in Russian].
2. A. S. Dmitriev, L. V. Kuz'min, A. I. Panas, et al., Usp. Sovrem. Radioélektron., No. 9, 40 (2003).
3. V. F. Kravchenko, O. V. Lazorenko, V. I. Pustovoĭt, and L. F. Chernogor, Dokl. Akad. Nauk **410**, 744 (2006).
4. V. F. Kravchenko, O. V. Lazorenko, V. I. Pustovoĭt, and L. F. Chernogor, Dokl. Phys. **52**, 129 (2007) [Dokl. Akad. Nauk **413**, 31 (2007)].
5. O. V. Lazorenko, S. V. Lazorenko, and L. F. Chernogor, ÉViÉS **9** (9), 31 (2004).
6. V. F. Kravchenko, O. V. Lazorenko, V. I. Pustovoĭt, and L. F. Chernogor, Dokl. Phys. **51**, 461 (2006) [Dokl. Akad. Nauk **410**, 38 (2006)].
7. V. F. Kravchenko and V. L. Rvachev, *Boolean Algebra. Atomic Functions and Wavelets for Physical Applications* (Fizmatlit, Moscow, 2006) [in Russian].
8. V. F. Kravchenko, *Lectures on Theory of Atomic Functions and Their Applications* (Radiotekhnika, Moscow, 2003) [in Russian].
9. E. G. Zelkin, V. F. Kravchenko, and V. I. Gusevskiĭ, *Constructive Methods of Approximation in Theory of Antennas* (Saĭns-Press, Moscow, 2005) [in Russian].
10. S. G. Mallat, *A Wavelet Tour of Signal Processing* (Academic, New York, 1998).
11. F. Auger, P. Flandrin, P. Goncalves, and O. Lemoine, *Time-Frequency Toolbox Reference Guide* (Rice Univ., Houstown, 2005).

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