ISSN 1028-3358, Doklady Physics, 2007, Vol. 52, No. 2, pp. 96–100. © Pleiades Publishing, Ltd., 2007.
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PHYSICS

Nonlinear Theory of Relativistic Cherenkov Generators Based on Irregular Waveguides with Allowance for Finite Wall Conductance

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Received July 10, 2006

PACS numbers: 41.60.Bq, 84.40.Ik
DOI: 10.1134/S1028335807020061

INTRODUCTION

There exist a number of modern mathematical models describing the action of powerful relativistic microwave devices involving irregular electrodynamic systems: relativistic Cherenkov generators of the running-wave lamp (RWL) type or inverse-wave lamp (IWL) type, gyrotrons [1], gyro-RWL [2], and gyrotons [3]. In these models, the excitation equations used are obtained under the boundary condition at metallic walls of an irregular waveguide in the transformed coordinate system of the form

$$[\boldsymbol{\rho}_0, \dot{\mathbf{E}}]\big|_{\rho = 1} = 0; \qquad (1)$$

where $\boldsymbol{\rho}_0$ is the normal to the surface of a regular cylinder. Condition (1) corresponds to the infinite wall conductance, which implies neglecting the ohmic loss in the electrodynamic system. A question arises about the adequacy of optimum variants obtained on the basis of these models, especially in the millimeter radio-wave region and in quasi-resonance regimes with a high diffraction Q-factor of the system. In the present study, this question is solved with respect to relativistic RWL and IWL systems and the solution is based on general excitation theory for irregular waveguides and allows for finite wall conductance.

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SELF-CONSISTENT NONLINEAR EQUATIONS FOR RELATIVISTIC CHERENKOV GENERATORS ON E_{0i}-MODES

We make use of the general excitation theory for irregular waveguides with allowance for a finite conductance of the wall, which was developed in [4, 5], and consider the case $n = 0$. Then,

$$\dot{E}_{\rho m} = -\sum_{i=1}^{I} \dot{A}_{mi}(z) J_1(\nu_{0i}\rho),$$

$$\dot{E}_{zm} = -\sum_{i=1}^{I} \dot{C}_{mi}(z) J_0(\nu_{0i}\rho),$$

$$\dot{B}_{\varphi m} = -j\sum_{i=1}^{I} \dot{V}_{mi}(z) J_1(\nu_{0i}\rho).$$

Using [4, 5] and the law of charge conservation, we arrive at the following dimensionless excitation equations:

$$\frac{d\dot{A}_{mi}}{dz} = (mW\dot{V}_{mi} + \nu_{0i}\dot{C}_{mi}) + (1-j)\times 2S_\sigma \frac{\sqrt{1+b'^2}}{b} \times \sum_k \dot{V}_{mk}\frac{J_1(\nu_{0k})}{J_1(\nu_{0i})},$$

$$\dot{C}_{mi} = -\frac{\nu_{0i}\dot{V}_{mi}}{mWb^2} + \frac{b'}{b}\left(-\frac{\dot{A}_{mi}}{\nu_{0i}} + \sum_{k\neq i}\frac{2\nu_{0i}}{\nu_{0k}^2 - \nu_{0i}^2}\frac{J_1(\nu_{0k})}{J_1(\nu_{0i})}\dot{A}_{mk}\right) - \frac{jG_0}{mWe_{0i}b^2}\frac{1}{N}\sum_{l=1}^{N} J_0\left(\nu_{0i}\frac{r_l}{b}\right)e^{-jmW\theta_l},$$

$$\frac{d\dot{V}_{mi}}{dz} = -mW\left\{\dot{A}_{mi} + b'^2\left[\dot{A}_{mi}\frac{1}{3}\left(1+\frac{4}{\nu_{0i}^2}\right) + \sum_{k\neq i}\frac{4(\nu_{0i}^2+\nu_{0k}^2)}{(\nu_{0i}^2-\nu_{0k}^2)^2}\frac{J_1(\nu_{0k})}{J_1(\nu_{0i})}\dot{A}_{mk}\right] - bb'\left(-\frac{\dot{C}_{mi}}{\nu_{0i}} + \sum_{k\neq i}\frac{2\nu_{0k}}{\nu_{0i}^2-\nu_{0k}^2}\frac{J_1(\nu_{0k})}{J_1(\nu_{0i})}\dot{C}_{mk}\right)\right\} + \frac{G_0}{e_{0i}b}\left(\frac{1}{N}\sum_{l=1}^{N}J_1\left(\nu_{0i}\frac{r_l}{b}\right)\left(\frac{\beta_{rl}}{\beta_{zl}} - \frac{r_l b'}{b}\right)\right)je^{-jmW\theta_l}. \quad (2)$$

Here, $\rho = \frac{r}{b(z)}$, $b(z)$ is the internal boundary of the irregular waveguide and $J_0(\nu_{0k}) = J_0(\nu_{0i}) = 0$.

The equations of motion for large particles are

$$\frac{dP_{rl}}{dz} = \frac{1}{\beta_{zl}}\left(\frac{\gamma_l\beta_{\varphi l}^2}{r_l} - E_{rl} - \beta_{\varphi l}F_z + \beta_z B_\varphi\right),$$

$$\frac{dP_{\varphi l}}{dz} = \frac{1}{\beta_{zl}}\left(-\frac{\gamma_l\beta_{rl}\beta_{\varphi l}}{r_l} - E_\varphi - \beta_z F_r + \beta_r F_z\right),$$

$$\frac{dP_{zl}}{dz} = \frac{1}{\beta_{zl}}(-E_z - \beta_r B_\varphi + \beta_\varphi F_r),$$

$$\frac{dr_l}{dz} = \frac{\beta_{rl}}{\beta_{zl}},\quad \frac{d\theta_l}{dz} = \frac{1}{\beta_{zl}},\quad \mathbf{P}_l = \gamma_l\boldsymbol{\beta}_l, \quad (3)$$

$$\gamma_l = \frac{1}{\sqrt{1-\boldsymbol{\beta}_l^2}} = \sqrt{1-P_{rl}^2-P_{\varphi l}^2-P_{zl}^2};$$

$$W\theta_l(0) = \frac{2\pi}{N}(l-0.5),\quad l = 1, 2, \ldots, N,$$

$$\beta_l(0) = \beta_0,\quad r_l(0) = r_0.$$

The expressions for physical high-frequency fields, which are written in terms of the calculated amplitudes with the space-charge fields taken into account, are of the form

$$E_r = -\frac{1}{b}\sum_{m=1}^{M}\sum_{i=1}^{I}J_1\left(\nu_{0i}\frac{r}{b}\right)\mathrm{Re}(\dot{A}_{mi}e^{jmW\theta}) - S_{qr},$$

$$E_\varphi = 0,$$

$$E_z = \mathrm{Re}\left[\sum_{m=1}^{M}\sum_{i=1}^{I}J_0\left(\nu_{0i}\frac{r}{b}\right)\dot{C}_{mi}e^{jmW\theta} + \frac{rb'}{b^2}\sum_{m=1}^{M}\sum_{i=1}^{I}J_1\left(\nu_{0i}\frac{r}{b}\right)\dot{A}_{mi}e^{jmW\theta}\right],$$

$$B_\varphi = \frac{1}{b}\sum_{m=1}^{M}\sum_{i=1}^{I}J_1\left(\nu_{0i}\frac{r}{b}\right)\mathrm{Re}(-j\dot{V}_{mi}e^{jmW\theta}).$$

The magnetostatic focusing field is the following:

$$F_r = -\frac{1}{2}rF_0'(z) + \frac{1}{16}r^3F_0'''(z),$$

$$F_z = F_0(z) - \frac{1}{4}r^2F_0''(z),\quad F_0 = \frac{B_0^r(z)e}{m_0\omega_0},$$

where $B_0^r(z)$ is the field along the axis.

The dimensionless parameters are

$$e_{0i} = 0.5J_1^2(\nu_{0i}),\quad G_0 = \frac{eI_0}{\pi\varepsilon_0 m_0 c^3},$$

$$S_\sigma = \frac{W_\sigma^0}{(1+j)\mu_a c} = \sqrt{\frac{\pi W}{\sigma\lambda_0\mu_a c}},\quad W_\sigma^0 = (1+j)\sqrt{\frac{\pi f\mu_\sigma}{\sigma}}.$$

Here, W_σ^0 is the wave resistance of the waveguide wall; μ_0 and σ are, respectively, the magnetic permeability and the specific conduction of the wall; f is the operation frequency; $W = \frac{f}{f_0}$; f_0 is the reference frequency (λ_0 is the wavelength (in a vacuum) corresponding to this frequency); c is the speed of light in a vacuum; ε_0 is the permittivity of free space; e and m_0 are, respectively, the electron charge and rest mass; and I_0 is the total current of the electron beam:

$$S_{qr} = \frac{G_0}{2r}\left(\frac{1}{\beta_{z0}} - \beta_{z0}\right).$$

We have taken the following relations between the basic dimensionless variables:

$$(r, z, b, L) = (r^r, z^r, b^r, L^r)\frac{\omega_0}{c},\quad W = \frac{\omega}{\omega_0},$$

$$\theta = \omega_0 t,\quad \boldsymbol{\beta}_l = \frac{\boldsymbol{\nu}_{el}}{c},$$

$$\mathbf{E} = \frac{\mathbf{E}^r}{E_m},\quad \mathbf{B} = \frac{\mathbf{B}^r c}{E_m},\quad E_m = \frac{m_0\omega_0 c}{e}.$$

We now can formulate the boundary conditions for the amplitudes $\dot{A}(z)$, $\dot{V}(z)$ in (2). We assume that for $z \le 0$ and $z \ge L$, the waveguide is regular. We denote the amplitudes in the forward and backward E_{0m} waves of the regular waveguide as $\dot{e}^{\pm}_{0mi}$ for $z \le 0$ and $\dot{e}^{\pm}_{Lmi}$ for $z \ge L$. Then, the general conditions for the amplitudes of propagating E_{0i} waves at the boundaries of the irregular-waveguide segment can be written in the form

$$\dot{A}_{mi}(0) = (\dot{e}^{+}_{0mi} - \dot{e}^{-}_{0mi})jk^{e}_{0i}, \quad \dot{V}_{mi}(0) = (\dot{e}^{+}_{0mi} + \dot{e}^{-}_{0mi})W,$$

$$\dot{A}_{mi}(L) = (\dot{e}^{+}_{Lmi} - \dot{e}^{-}_{Lmi})jk^{e}_{0i}, \quad \dot{V}_{mi}(L) = (\dot{e}^{+}_{0mi} + \dot{e}^{-}_{0mi})W, \quad (4)$$

$$k^{e}_{0i} = \sqrt{1 - \left(\frac{\nu_{0i}}{b}\right)^2}.$$

We note that for the correct formulation of the problem (2), it is sufficient to choose only two of four equations (4). While mathematically simulating various devices, at the input of the electromotive force, the amplitude $\dot{e}^{+}_{0mi}$ of the incoming wave is usually controlled (is given). At the output, the amplitude $\dot{e}^{-}_{Lmi}$ of the counterrunning wave is controlled (under the validity of the matching condition $\dot{e}^{-}_{Lmi} = 0$). If we exclude $\dot{e}^{-}_{Lmi}$ or $\dot{e}^{+}_{0mi}$ from (4), then the boundary conditions for the amplitudes of propagating E_0 waves can be written out in the following form:

$$\begin{aligned} W\dot{A}_{mi}(0) + jk^{e}_{0i}\dot{V}_{mi}(0) &= jk^{e}_{0i}W \times 2\dot{e}^{+}_{0mi}, \\ -W\dot{A}_{mi}(L) + jk^{e}_{0i}\dot{V}_{mi}(L) &= jk^{e}_{0i}W \times 2\dot{e}^{-}_{Lmi}. \end{aligned} \quad (5)$$

These relationships can be applied to determine the amplitudes of the direct and counterrunning waves in regular segments of the waveguide.

The boundary conditions for the amplitudes of supercritical E_{0i} waves are of the form

$$\begin{aligned} W\dot{A}_{mi}(0) + k^{e}_{0i}\dot{V}_{mi}(0) &= 0, \\ -W\dot{A}_{mi}(L) + k^{e}_{0i}\dot{V}_{mi}(L) &= 0. \end{aligned} \quad (6)$$

Physical conditions (6) correspond to the attenuation of supercritical waves with distance from the boundaries of the irregular waveguide segment. Expressed in the chosen variables, the dimensionless power carried by the wave field through the waveguide cross section is

$$P(z) = \sum_{mi} e_{0i} \times \mathrm{Im}\left[\dot{A}_{mi}(z)\overset{*}{\dot{V}}_{mi}(z)\right].$$

In regular segments, and also at the waveguide points, where $b' = 0$, the powers (in terms of the chosen dimensionless variables) of the forward and backward waves are expressed as

$$P^{\pm} = \sum_{i} e_{0i} \times \mathrm{Im}\left[\left(\dot{A}_{mi} \pm \frac{j}{k^{e}_{0i}}\frac{d\dot{A}_{mi}}{dz}\right)\left(\dot{V}_{mi} \pm \frac{j}{k^{e}_{0i}}\frac{d\dot{V}_{mi}}{dz}\right)^{*}\right].$$

The interaction efficiency is estimated according to the wave efficiency, i.e., by the ratio of the power of the carried electromagnetic wave through the cross sections z of the waveguide segment $[0z]$ to the electron-beam power

$$\eta_{\nu mi} = \frac{\dot{\mathrm{I}}\mathrm{m}(\dot{A}_{mi}(z)\dot{V}^{*}_{mi}(z)) - \dot{\mathrm{I}}\mathrm{m}(\dot{A}_{mi}(0)\dot{V}^{*}_{mi}(0))}{(\gamma_0 - 1)\dfrac{G_0}{e_{0i}}}. \quad (7)$$

The electron efficiency is used for verification of the accuracy and is calculated by the following method:

$$\eta_e = \frac{1}{N}\sum_{l=1}^{N}\frac{\gamma_0 - \gamma_l(z)}{\gamma_0 - 1}, \quad \gamma_0 = \frac{1}{\sqrt{1 - \boldsymbol{\beta}_0^2}}.$$

The features of the interaction are demonstrated by the grouping function proportional to the electric-current amplitude of the sth harmonics in the modulated electron beam:

$$G_r = \left(\frac{1}{N}\sum_{l=1}^{N}[\sin(mW\theta_l) + \cos(mW\theta_l)]\right)^{1/2}.$$

The profile of the irregular corrugated waveguide is given in the form

$$b(T) = b_0 + h_\nu(T)\sin^2[n_\nu\pi(T + D_\nu(T))], \quad (8)$$

where $T = \dfrac{z - z_0}{L_\nu}$, z_0 and L_ν are the beginning and the length of the irregular segment, n_ν is the number of periods, $h_\nu(T)$ is the corrugation depth, $D_\nu(T)$ is the function defining the period, $D_\nu(0) = 0$, $D_\nu(1) = 0$; for $D_\nu(T) = 0$, the period is constant and equals $d = \dfrac{k_0 L_\nu}{n_\nu}$ in the chosen units. The relationships $h_\nu(T)$ and $D_\nu(T)$ were approximated by expansions in terms of shifts of a stan-

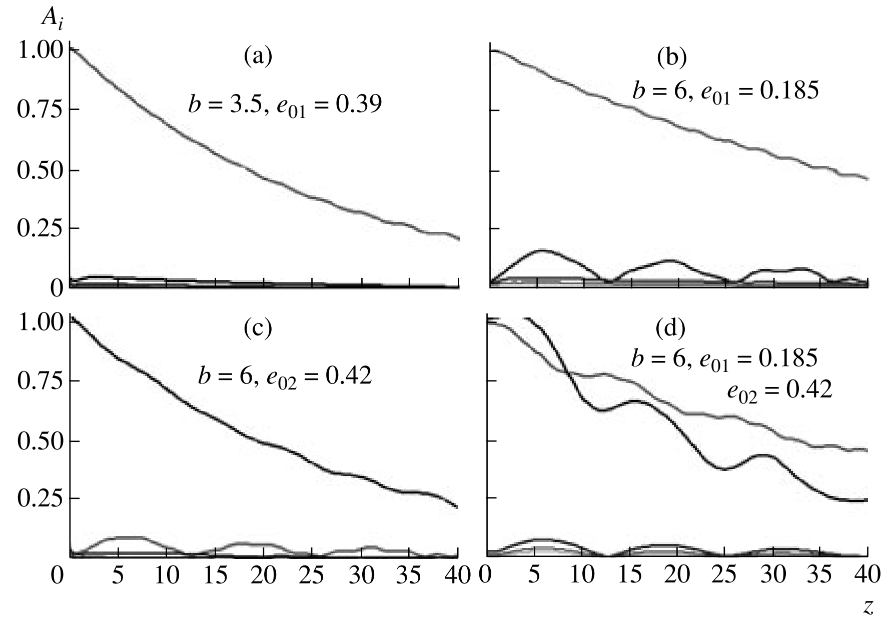

Fig. 1. Calculated results for the attenuation of E_{01} waves for different calculation parameters: b_0, e_{01}, and e_{02}.

dard finite function $\varphi_3(x)$ that corresponds to the B spline of the third order

$$h_\nu(T) = \sum_{k=1}^{K} h_k^\nu \varphi_3[T(K-3) - k + 2],$$

$$D_\nu(T) = \sum_{k=1}^{K} d_k^\nu \varphi_3[T(K+3) - k - 1], \qquad (9)$$

$$\varphi_3(x) = \begin{cases} 0, \quad |x| \geq 2, \quad \dfrac{(2-x)^3}{6}, \quad 1 \leq x \leq 2; \\ \dfrac{1}{6}[1 + 3(1-x) + 3(1-x)^2 - 3(1-x)^3], \\ 0 \leq x \leq 1; \\ \varphi_3(-x), \quad x \leq 0. \end{cases}$$

We can note that, in this approximation, the values of the coefficients h_k and d_k coincide with those of the functions $h_\nu\left(\frac{k-2}{K-3}\right)$ and $D_\nu\left(\frac{k+1}{K+3}\right)$, respectively.

ANALYSIS OF THE RESULTS

Prior to beginning the verification of optimal variants of a relativistic RWL and IWL, which were obtained previously without allowance for the loss in walls of an electrodynamic system, it is necessary to test the set of excitation equations obtained. We can realize this task using the classical attenuation theory for E_{0i} waves in regular waveguides [6]. In this case, it is natural to pay attention to the following fact. We need to distinguish the concepts of proper waves and normal waves of a regular waveguide. The proper waves correspond to the particular solutions of Maxwell equations beyond the sources. These solutions satisfy the approximate Shchukin–Leontovich boundary conditions on the waveguide walls. The normal waves correspond to the particular solutions obtained under the validity of condition (1) on waveguide walls. These solutions are presented in the set of excitation equations obtained by us. As was shown in [7, 8], the proper waves are energy independent. The normal waves in a waveguide with a finite wall conductance turn out to be coupled, which follows from both the general theory [4, 5] and the specified theory for E_{0i} waves. In the classical literature on electrodynamics, the attenuation of normal waves is considered as that of isolated waves, which is, generally speaking, incorrect. For the dominant E_{01} wave and for the waveguide radius and the operating frequency corresponding to the supercriticality conditions for E_{0i} waves ($i \geq 2$), this approximation is assumed to be admissible.

The attenuation coefficient for the E_{01} wave, which is calculated in this approximation, can serve as a milestone in verifying the set of Eqs. (2) for $G_0 = 0$ and $b = b_0 = \text{const}$.

Figure 1a presents the calculation results in the case of the attenuation of the E_{01} wave for $b_0 = 3.5$ and $\lambda = 3.2$ cm. (To amplify the effect of the boundary impedance in the given calculations, the value of σ is decreased down to $\sigma = 30$ S/m compared to the value for copper $\sigma = 5.6 \times 10^7$ S/m.) The waveguide is matched at its right end, whereas $e_{01}^+ = 0.39$, $e_{0i>1}^+ = 0$

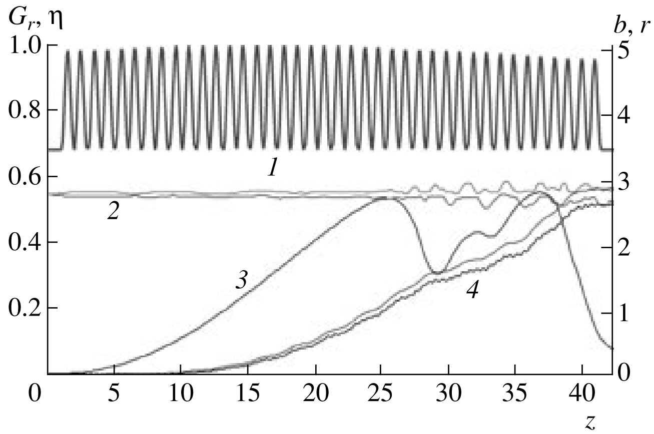

Fig. 2. Characteristics of the variant of a long running-wave lamp (long RWL) with an irregular corrugation. (*1*) b, (*2*) $r_{0\min}$, $r_{0\max}$, (*3*) G_r, (*4*) η_e, η_v.

at its left end. As is seen from Fig. 1a, the amplitudes of the supercritical E_{02} and E_{03} waves excited in the waveguide are very small. Therefore, the attenuation coefficient calculated according to expression (2) virtually coincides with that given in [6] for the parameters under consideration, $\alpha_{01} = \frac{S_\sigma}{b_0^r k_{01}^e} = 0.079\ \text{cm}^{-1}$. The characteristic of the variant with $b_0 = 6$, $e_{01}^+ = 0.185$, and $e_{0i>1}^+ = 0$ for the same values of λ, σ are presented in Fig. 1b. The E_{02} wave now is propagating.

As is seen from Fig. 2, the E_{02} wave is periodically excited due to its coupling with the E_{01} wave. The periodicity of the E_{02}-wave excitation is associated with the difference in the phase velocities of the E_{01} and E_{02} waves. In Fig. 1c, the calculation results are presented for the variant $b_0 = 6$, $e_{01}^+ = 0$, $e_{02}^+ = 0.42$ with the same other parameters. Now, at the left end of the waveguide segment, the E_{02} wave excites the basic E_{01} wave. This excitation also has a periodic nature, which is associated with the periodicity of the energy transformation from E_{02} to E_{01} and vice versa due to the difference in their phase velocities. In Fig. 1d, the results are shown for the variant with $b_0 = 6$ and the same input powers of the E_{01} and E_{02} waves: $e_{01}^+ = 0.185$ and $e_{02}^+ = 0.42$.

Here, the transformation effect is expressed much more strongly (the excitation of the supercritical normal E_{03}, E_{04}, E_{05}, and E_{06} modes should also be taken into account). In order to clarify the effect of the finite wall conductance, variants of calculations are performed for generators and amplifiers [4] with allowance for the wall loss. It turned out that while using an electrodynamic system in the form of a corrugated-waveguide segment with copper walls ($\sigma = 5.6 \times 10^7$ S/m) in devices with the operation frequency $f <$ 10 GHz, the ohmic loss does not exceed 1% of the power generated and the contribution of this effect lies within the calculation accuracy.

For $f = 100$ GHz, the ohmic loss attains 3–4%. In Fig. 2, the characteristics of the variant of a long RWL with an irregular corrugation are presented for the following parameters: $\beta = 0.9$, $I_0 = 510$ A, $r_0 = 3.8$, $\lambda_0 = 2$ mm ($f = 150$ GHz), $L_v = 39.26$, $n_v = 40$, $b_0 = 3.49$, $\eta_e = 56.5$, $\eta_v = 51.6$, $D_v = 0$, $h_1^v = 1.386$, $h_2^v = 1.547$, $h_3^v = 1.724$, $h_4^v = 1.337$, and $h_5^v = 0.575$. The effect of the ohmic loss is manifested as splitting of curves of the wave and electron efficiencies. The difference $\eta_e - \eta_v$ corresponds to the relative value of lost power.

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Translated by G. Merzon